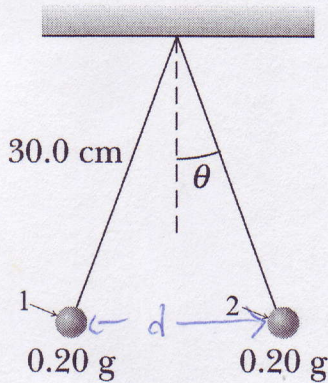


Midterm (24 points)

Multiple Choice: Circle the best answer for each of the three multiple choice questions.

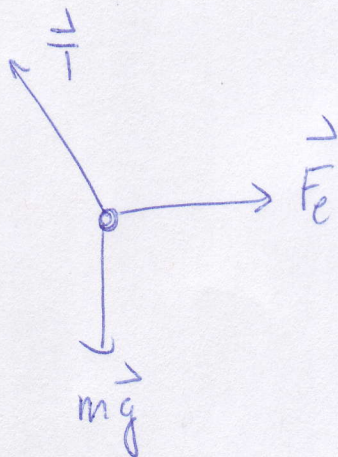
- (2pts) 1. Two identical spheres (same radius and mass) are suspended from strings of equal length as shown in the figure. Initially, both spheres carry a charge of q and the electrostatic repulsion causes the string on the right to make an angle θ with respect to the vertical dashed line.



If the charge on sphere 1 is changed to $q/2$ and the charge on sphere 2 is changed to $2q$, what happens to θ ?

- (a) θ increases.
- (b) θ decreases.
- (c) θ stays the same.
- (d) More information is needed.

FBD of 2.



initially $F_e = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q}{d^2} \sim \frac{q^2}{d^2}$

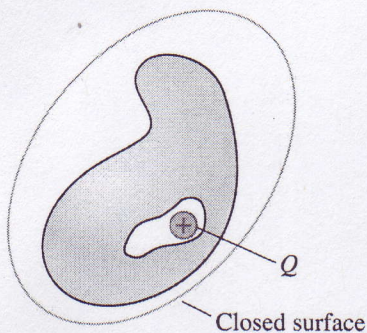
after charge changed.

$F_e = \frac{1}{4\pi\epsilon_0} \frac{\frac{q}{2} \cdot 2q}{d^2} \sim \frac{q^2}{d^2}$

same.

10:01
01:11

- (2pts) 2. The figure shows a hollow cavity within a neutral conductor. Point charge Q is inside the cavity. What is the net electric flux through the closed surface that surrounds the conductor?

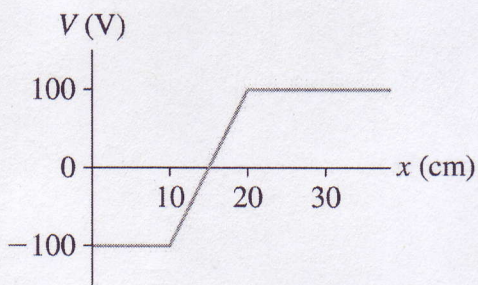


$$\Phi_e = \frac{Q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$Q_{in} = Q$ since conductor is neutral.

- (a) $-2Q/\epsilon_0$ (b) $-Q/\epsilon_0$ (c) zero (d) Q/ϵ_0 (e) $2Q/\epsilon_0$

- (2pts) 3. The electric potential along the x -axis of a coordinate system is given by the plot below. What is the electric field along the x -direction at the position $x = 15$ cm?



$$\vec{E}_x = - \frac{dV}{dx} = - \text{slope}$$

- (a) $-2000 \frac{V}{m} \hat{i}$ (b) $-20 \frac{V}{m} \hat{i}$ (c) zero (d) $+20 \frac{V}{m} \hat{i}$ (e) $+2000 \frac{V}{m} \hat{i}$

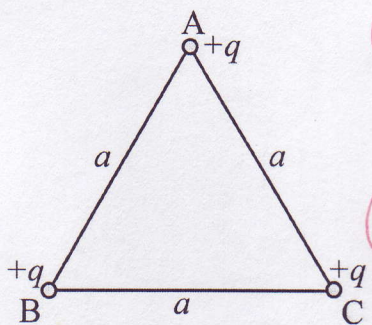
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{200 \text{ V}}{10 \text{ cm}} = \frac{200 \text{ V}}{0.10 \text{ m}} = 2000 \frac{V}{m}$$

$$\therefore \vec{E}_x = -2000 \frac{V}{m}$$

$$\vec{E} = -2000 \frac{V}{m} \hat{i}$$

Free Response: Write out complete answers to the following questions. Show your work since it allows us to be generous with partial credit.

- (6pts) 4. Three identical charges of mass m and charge q are held in place at the corners of an equilateral triangle. The charges are then simultaneously released from rest. With what speed v_f do the charges move once they are very far apart? Find an expression for v_f in terms of q , m , a , and Coulomb's constant K .



$$\textcircled{1} \quad \Delta K + \Delta U = 0. \quad \textcircled{1}$$

$$K_i = 0$$

$$\textcircled{1} \quad K_f = 3 \frac{1}{2} m v_f^2$$

↑
three charges.

$U_f = 0$ since
charges very
far apart.

$$U_i = U_{AB} + U_{AC} + U_{BC} \quad (\text{three pairs of charges}).$$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$$

$\textcircled{2}$

\therefore from $\textcircled{1}$

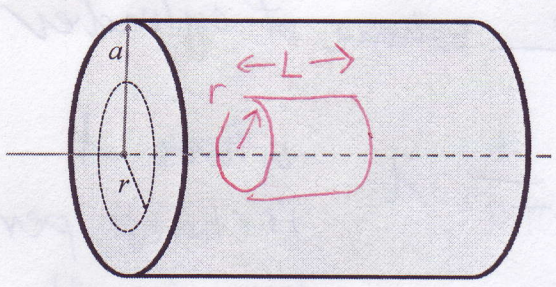
$$\frac{3}{2} m v_f^2 - \frac{3Kq^2}{a} = 0$$

$$\therefore v_f^2 = \frac{2Kq^2}{ma}$$

$$v_f = \sqrt{\frac{2Kq^2}{ma}}$$

$\textcircled{2}$

(6pts) 5. A section of a long cylinder of radius a is shown below. The cylinder has a *uniform* charge per unit volume ρ . Find an expression for the magnitude of the electric field E at point *inside* the cylinder that is a distance r from the cylinder axis.



Gauss's law

By symmetry \vec{E} is radial.

① $\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$

Choose Gaussian surface to be cylinder concentric with charged cylinder a contained within charged cylinder. Gaussian surface has volume $\pi r^2 L = V$

$\therefore Q_{\text{in}} = \rho V = \rho \pi r^2 L$ ①

$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_{\text{curved}} \vec{E} \cdot d\vec{A} + \int_{\text{sides}} \vec{E} \cdot d\vec{A}$ ①

since $d\vec{A} \perp \vec{E}$ for left & right side pieces.

$= \int_{\text{curved}} E dA = E \int_{\text{curved}} dA = E \underbrace{2\pi r L}_{\text{surface area of curved part of Gaussian surface}}$ ①

$\therefore E \cancel{2\pi r L} = \frac{\rho \pi r^2 L}{\epsilon_0}$

$\therefore E = \frac{\rho r}{2\epsilon_0}$

magnitude of \vec{E} ②
inside uniformly charged cylinder.

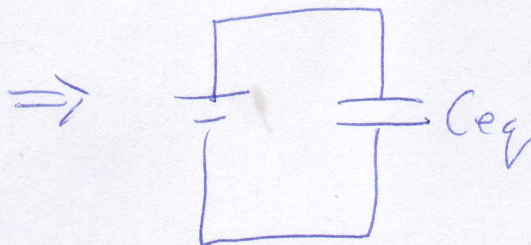
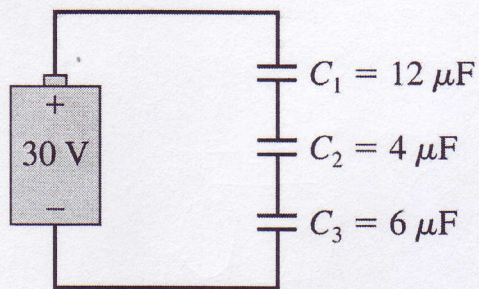
Note: $\rho = \frac{Q}{\pi a^2 L}$ ← total charge of cylinder
 ← volume of cylinder

$= \frac{1}{\pi a^2} \frac{Q}{L} = \frac{1}{\pi a^2} \lambda$ where λ is charge per unit length.

$\therefore E = \frac{\rho r}{2\epsilon_0} = \frac{\lambda r}{2\pi\epsilon_0 a^2}$

also acceptable.

$\therefore E = \frac{\rho r}{2\epsilon_0}$

(6pts) 6. What is the charge on capacitor C_3 ?

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_{eq} = 2.0 \mu\text{F}$$

$$C = \frac{Q}{\Delta V} \quad \textcircled{1} \quad \therefore Q_{eq} = C_{eq} \Delta V = (2.0 \mu\text{F})(30\text{V}) = 60.0 \mu\text{C} \quad \textcircled{1}$$

$Q_1 = Q_2 = Q_3 = Q_{eq}$ for series capacitors.

$$\therefore \boxed{Q_3 = 60.0 \mu\text{C}}$$